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Mathematics News Letter

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Published in the interest of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics

A Challenge to Forward-looking Mathematics Teachers in the Colleges and High Schools of Louisiana and Mississippi.

VOL. 2 BATON ROUGE, LA., APRIL, 1928 No. 6

[last no. of this vol]

NEW STATUS OF THE MATHEMATICS NEWS LETTER

At the recent annual joint meeting of the Louisiana-Mississippi Section of M. A. of A. and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics, the following provisions concerning the Mathematics News Letter were adopted by these organizations:

1. Eight issues of the Letter are to be published during the year.

2. The subscription price for each new subscriber shall be one dollar per year.

3. The executive committees of the Section and Council, acting jointly, are authorized to provide for organized effort in every county of Mississippi and parish of Louisiana to secure a maximum number of paid-up subscriptions.

4. Since the News Letter is designed to promote the joint interests of college and high school mathematical workers, a high school mathematics teacher will assist a college mathematics teacher in the task of editing and managing the Letter.

5. Professor S. T. Sanders, of the Louisiana State University, and Mr. Henry Schroeder, of the Ruston, Louisiana, High School will do the editing work.

6. The Letter will be printed by the Louisiana Polytechnic Institute, at Ruston, Louisiana, will be mailed out through the

Ruston post-office, under the direction of the newly elected secretary-treasurer, namely, Professor Jas. P. Cole, head of the L. P. I. mathematics department.

It is the earnest desire of the editors to make the News Letter as effective as possible. Such an ideal can be approached only through the united efforts of many minds inspired by a vision of higher levels of public interest in mathematics, a more deepened general appreciation of its educational values. In harmony with this conception of our responsibility we have thought it well to publish a list of topics, or topic fields, which the editors propose to have discussed, or made the subject of carefully prepared papers to be furnished by competent persons.

Readers who have in mind subjects, or fields of subject matter, not placed in the list here offered, are urged to communicate with us concerning such subjects. More than this, they are invited to send us any results of their own experience or investigations, which, manifestly, would be of value to any class of mathematical workers.

Topics 24-30 were furnished by several different individuals in response to the editorial request for subjects to be treated.

NEWS LETTER TOPICS

1. Articles dealing with what may broadly be described as the correlation of high school and college mathematics courses. Under this head systematic effort will be made to publish in clear and popular style constructive discussions of

(a) Improvement in schemes of articulation between secondary and college mathematical courses and programs.

(b) Comparative studies of these schemes as they now exist in different states of the Union.

(c) How high school mathematics programs may furnish a maximum service to colleges.

(d) How college programs in mathematics may best serve the high schools.

(e) Comparative study of college and high school objectives mathematics teaching.

2. Sectioning classes in mathematics according to the different grades of ability.

3. Mathematics in its relation to the so-called social sciences.

4. Mathematics and the principle of election in a liberal arts college program.

5. The mathematical mind and the non-mathematical mind —is the difference real?

6. Methods of motivating interest in mathematics.

7. History of mathematics and its applications.

8. Sketches of successful mathematicians.

9. Mathematics and school administrations.

10. Mathematics as culture.

11. Mathematics as discipline.

12. Mathematics as a tool.

13. Personality in mathematics teaching.

14. Problem solving department.

15. Review of important articles in the Mathematics Teacher.

16. Review of important articles in the American Mathematical Monthly.

17. M. A. of A. programs and projects.

18. Programs and projects of the National Council of Teachers of Mathematics.

19. Promoting the annual meeting of Louisiana-Mississippi Section and Council.

20. News notes from college and high school mathematics departments of Louisiana and Mississippi.

21. Increasing the News Letter Subscriptions.

22. The work of the National Committee on Mathematical Requirements.

23. Mathematical papers from college and secondary teachers.

24. Correlation between mathematics grades and grades in other subjects.

25. Tests for achievement and knowledge.

26. Drill and its importance.

27. Methods of instruction in mathematics that have been successful.

28. Place of the mathematics club in high school and college.

29. Comparison of results from "general mathematics" courses with results of courses separated into branches, as algebra, geometry, etc.

30. Euclidean college geometry.

CUTTING DOWN OPERATING EXPENSE

It is our hope that future issues of the News Letter shall be admitted to the mails as second class matter. If this is done, a very material reduction in the cost of sending it out will have been secured. On the other hand, under the postal rules, only a small number of copies of the Letter can be mailed as second class matter to those who are not regular paid-up subscribers. Because of this necessity *we take occasion to announce to all present readers of the News Letter that those who desire to continue to receive it each month should at once send in to Secretary Cole, L. P. I., Ruston, Louisiana, the price of a year's subscription, or half year, unless they are already listed with the paid-up subscribers.*

In this connection it should be definitely understood that those who subscribed under the former terms, namely, fifty cents a year will continue to receive the Letter until their subscription period has expired, the dollar-a-year terms applying merely to new subscribers.

ACADEMY OF SCIENCES AND LOUISIANA-MISSISSIPPI SECTION OF M. A. OF A. TO MEET CONJOINTLY IN LAFAYETTE IN 1929.

In a note to the editors, President I. Maizlish, of the Louisiana Academy of Sciences says:

"At the first annual meeting of the Louisiana Academy of Sciences, which, as you know, was held at Louisiana College last Saturday, it was voted that we have our next meeting at Lafayette conjointly with the Louisiana-Mississippi Section of the Mathematical Association of America.

"May I ask that you be good enough to insert a notice to this effect in the MATHEMATICS NEWS LETTER?

MATHEMATICS AN AID TO LANGUAGE

(From an address by Geo. B. Olds, President Emeritus of Amherst College, as published in the April Mathematics Teacher.)

"Another form of training given by mathematics not often

thought of is that of clarity and directness of expression. It might seem to the layman a long way from mathematics to the use of one's vernacular, and yet there is a most marked interdependence. A number of years ago I had a pupil at Amherst, now at the head of the English Department of one of our best eastern colleges, and one of the most brilliant teachers of English in the country. He has, also, a style which is an inspiration to his pupils, a daily example of what is best in oral and written expression. At college he took the first required year in mathematics and elected calculus in the second year. At that time he had made up his life to the study and teaching of English. I assumed, of course, that he would take no more mathematics and was surprised to learn from a colleague that he was determined to continue the subject until the end of his college course. When I asked him his reason he said that he wished the kind of training that mathematics gave in order to do two things: one, to prune, and the other to clarify his English style. He felt that he was too diffuse, too exuberant in his way of expressing himself, and too obscure, at times. Through the close reasoning and clear expression demanded by courses in mathematics he hoped to effect a cure. That he was satisfied in the end with the choice he had made became evident, when, some years later, he wrote an article recounting his experiences in this study of an abstract science, recommending such a course to those who suffered as he thought he suffered in the use of the English language."

MATHEMATICS BASIC IN EVERY FIELD

Every field of endeavor has its problems. Progress in the field hinges upon the ability of the worker to solve the problems. Advance in the field of agriculture is conditioned upon the solution of problems of adjustment between seed and soils, markets and transportation routes. Improvement in government, from the small village to the huge nation, depends upon the capacities of department heads to install executive machinery adapted to secure with certainty the predetermined objectives. Industrial plants employ experts to point out the *certain* ways in which both output and market can be enlarged. Sources of efficiency are analyzed, measured, and registered with the definite view of eliminating from processes employed every element of hazard

and substituting therefor a factor, the implications of which may be inferred with *mathematical certainty*. One needs but to glance at such a book as Durell's "Fundamental Sources of Efficiency" to get an insight into the truly mathematical modes underlying methods of efficiency-measurement now being installed in every field of corporate industry. The "science of necessary conclusions" is at the very heart of such concepts as "Unit and Multiplier," "Multiplication Groups," "Orders of Material," "Symbolism," "Kinematic and Dynamic," "Error and Paradox," "Combinations of Efficients," Categories"; or in such questions as, "State some of the advantages of having two engines in a factory instead of a very large one." "Why does it pay a railroad system to develop its trunk lines more thoroughly than its branch lines?" "State the efficiences connected with the use of one comprehensive electric light system instead of several thousand oil lamps." "Why should goods when bought on a large scale cost less?" All these questions and scores of others which might be quoted indicate answers which must take the form of the necessary conclusion—answers yielded with the same closed-circuit click that goes with a mathematical deduction. They point unerringly to the fact that in all circles of world action the leaders are reaching out for their proposed objectives by the same procedure of mind as that by which solutions of mathematical problems are obtained. From a priori considerations this should be so. Precision of results in any given operation inevitably leads to measurement, comparison, ratio.

WASTED TIME IN HIGH SCHOOL ALGEBRA

By MARY CAMPBELL
South Park College, Texas

When one considers the amount of interesting and important nathematical material which might be presented to high school udents and the absolute impossibility of finding time for all it, he can not fail to be impressed with the idea that it is well-nigh criminal to waste any of the precious hours alloted to mathematics. In my years as high school teacher and as a supervisor of high school teaching, I was constantly seeing evi-ence of wasted time; and I came to list the failure to make

good use of time as one of the most common faults of mathematics teachers. Since I have been doing college teaching and dealing with the results of high school teaching, I am still convinced that much time is being poorly invested during the high school years.

One particular topic in algebra which nearly always occupies more time than is necessary is the topic of factoring. Factoring is undoubtedly a subject of great importance; but it is important because of what it enables us to do. It is always a means to an end, never an end in itself. Recently I asked one hundred college freshmen the question: "why do we learn to factor?" Out of the hundred only two ventured an answer and both of these said the same thing: "Factoring makes some problems easier." Neither could make his answer more definite.

Would not this vagueness be removed if we presented the subject differently? After all, why do we factor? One reason is that we may be able to reduce fractions. Very well, then, as soon as we have learned to do one type of factoring, we are ready to begin reducing fractions. Factoring is useful in addition, subtraction, multiplication, and division of fractions. If these uses of factoring are explained to the students, is there any reason why factoring should not be learned at the same time that the student is acquiring facility in handling fractions? I have experimented on several classes and I have found that students can learn factoring and addition, subtraction, multiplication and division of fractions in exactly the time needed to provide adequate drill on factoring alone. And the cheering fact is that they learn the factoring better than they do when it is taught alone.

Another word that I'd like to say about factoring is that the simple problems are almost always worth more time than the complicated ones. It is interesting to a student to unravel a long problem; but, for his future good, he should become letter perfect in factoring forms like $(8x^3—27y^3)$ before he attempts one like: $2x^2—2ay+2ax—4xy+2xy^2$. If there isn't time for both, omit the latter.

Other ways of avoiding waste of time will occur to the alert teacher. In speaking of factoring, I have not attempted to find all the "lost motion" of algebra. I have aimed only at the thing that I consider the worst offender.

A STUDY OF RECORDS OF STUDENTS ENTERING L. S. U. WITH ONLY ONE UNIT OF HIGH SCHOOL ALGEBRA

H. L. GARRETT
Professor of Secondary Education Louisiana State University

Have students who offered for admission to the University only one unit of elementary algebra succeeded in college mathematics? What has happened to such students in the College of Engineering? Are there significant facts to be noted in the records of these students taken as a group?

To answer questions such as the above, the writer has studied the records of 208 students, including 151 men and 57 women, who were admitted to the University during the past fifteen years with one unit of high-school algebra. The current enrollment was not included. Also, students who attended the University under emergent conditions, such as the S. A. T. C. of 1918, were not included.

With regard to high school origin and distribution among the colleges of the University, the main facts for the 208 students are summarized below:

Graduates of public high schools in Louisiana	181
Graduates of private secondary schools	12
Graduates of high schools in other states	15
Number registering in College of Arts and Science	120
Number registering in the College of Engineering	50
Number registering in the College of Agriculture	18
Number registering in Teachers College (Home Ec.)	20
Number admitted prior to 1922, 253; in 1922 or later	183

The principal facts concerning the college records of these 208 students are summarized below:

Number taking no mathematics in freshman year	62
Number passing college algebra (Math. 1)	44
Number failing in college algebra	79
Number eliminated because of low grades	35
Number resigning with no record established	35
Number resigning with very poor record	17
Number passing all courses in College of Engr.	0
Number passing all freshman courses: with mathematics, 8; without mathematics, 5	13

Number remaining through freshman who passed
not more than one-half of work scheduled............ 38
More than half but not all of work............................ 68

The figures presented in the above summary do not check with the total because the Registrar's office had, in many cases, very meager and incomplete data to record.

Certain significant facts may be noted in the records of these 208 students. (1) 78 students, more than one-third, were eliminated outright or resigned, leaving a very poor record or no record at all. (2) A large group, 62 students, avoided mathematics in the freshman year. (3) Of the number taking mathematics, about one-third succeeded in passing college algebra. (4) Only six per cent of the total number passed all freshman courses. (5) No student in engineering made a clear record in the freshman year. Only two passed all mathematics courses.

This study should raise certain questions which ought to receive serious consideration by both high school and college people. Is there not evidence to support the assumption that pupils who deliberately avoid mathematics in high school are likely to be poor college students in any field? Would not the College of Engineering be justified in barring any freshman who has not had adequate high-school training in mathematics? Would it not be helpful to high school faculties in counseling pupils who expect to enter college to have information concerning the experience of the higher institutions with certain types of students and varying degrees of preparation?

MORE TEACHING OF THE FUNDAMENTALS NEEDED IN ALGEBRA

By ALICE KNIGHTON
Baton Rouge High School

In the "Mathematics News Letter" for December, Dr. Webber of L. S. U. told the story of the evolution of freshman mathematics within the last forty years. With apologies to Dr. Webber, I should like to give my views of high school mathematics in the last twenty years.

A glance at our present text-books shows problems of a simpler, more practical type than those of twenty years ago; the "fancy problems" are omitted. This I consider an improve-

ment. A high school pupil should spend the limited time that he has in getting fundamentals well fixed, rather than in learning to solve the more elaborate types.

My experience has been that it is a lack of knowledge of fundamentals that accounts for the failure of pupils to master the higher types. Children reach their senior year in high school without having a working knowledge of fractions. They cancel in addition of fractions and when told to take one-half of the coefficient of x in completing the square, if that coefficient is a fraction, they must be taught to perform that operation. This frequently happens. On a recent examination in eleventh grade algebra, several pupils gave this expression as a final answer: $\frac{2\sqrt{a}+30}{6}=2\sqrt{a}+5$. These pupils had been taught the correct form, special attention had been called to the incorrectness of such a form as this, and many were made to work after school. Yet such results as above were received.

Granting that there is nothing wrong with our texts, what is wrong with the high schools? Why so many failures in college freshman mathematics? Is it the teacher or the pupil?

We use the study-recitation method in our teaching. We spend a few minutes at the beginning of each recitation period in straightening out unforeseen difficulties encountered in working out the previous assignment; then, a test based on the assignment is given; papers are graded and a new assignment made; this is the real teaching part of the period, and, lastly, work is begun by pupils on the assigned work—to be completed at home. This is given as a means of drill and must be carefully checked or there will be no home work.

Do we give too much help in the assignment? Time after time I have tried to give a minimum amount of help, hoping to teach the child to use his own method of attack, to arouse his reasoning powers. But pupils return with problems unsuccessfully attempted. The majority of high school children will not attempt to solve a problem more than twice and often give up with the first effort. I remember, as a high school pupil, my aim was to "dig" until the problem was solved regardless of the number of attempts or time used. The result was a mastery of the subject.

After getting actual statistics on activities of school children of Baton Rouge, we found that a large percentage attend picture shows and other places of amusement from three to five times

a week. With so much outside attraction, there is little interest in school work and an inferior product is the result. Hard working conscientious teachers do all they can to incite children to their best. We follow them eagerly in their work after they leave high school, and are so often disappointed in the results obtained in the freshman year in college.

Children do not stop long enough, or disengage themselves from the outside world, which parents permit them to enter at all times, to "digest" what they get in the short recitation period. The result is that principles are not fixed, a smattering is obtained which lasts for a short time only; thus failures in college mathematics.

I have found that by giving one year of algebra in the last year of high school, instead of the required half year, the pupil is strengthened wonderfully. Often the work is practically a repetition of what came before—to some it seems almost new—still, the principles are better fixed in the minds of the pupils than ever before.

Not every high school pupil is the disinterested one that I have pictured, but a large percentage are, and they are our problems, not the bright ones. What shall we do to help the cause?

IS MATHEMATICS CULTURAL?

By C. H. BEAN
Professor of Psychology, Louisiana State University

That mathematics has practical value cannot be questioned, and to the specialist in mathematics, there may be no doubt about its cultural effects. It has occupied an enviable place in high school and college courses long enough to convince its friends that it wears royal headgear by divine right. This article is written in defense of its title against seditious mutterings heard today.

Before mathematics can be defended as a means to this end, culture must be adequately defined. Most of the definitions of culture, instead of illuminating that idea carry it further into the dark. Even those of us who have studied mathematics much and with zeal have discovered no consequences that are too abstract, too wonderful, too supernatural to be identified, defined, even measured.

Several investigators in my field won for us no little disrepute among teachers of ancient languages and of mathematics by finding experimentally that there is only a small amount of unconscious transference of culture or training from one subject to another. Let it be remembered that every one of these investigators was himself a specialist either in the ancient languages or in mathematics as well as in psychology, which accounts for the fact that every one of them expected and hoped to find a large accumulation of "general culture" at the end of a course in either of these fields-culture that would be fundamental to all other subjects, and that would make them easier to master. All other psychologists were disappointed too, as they always are when they fail to find means of making life's tasks easier. There are grave reasons for doubt, therefore, about culture as a "mystical, unfathomable somewhat." But this does not preclude the scientific isolation of more tangible cultural effects of mathematical training.

Anything that is cultural is beneficial to the individual, educative. But nothing is recognized in pedagogical circles today as educative that does not make a person more able to adapt himself to his environment. Part of environment is physical and the rest is social. Knowledge of mathematics aids in adaptations primarily to physical objects, and secondarily to personalities. If the concept culture is limited to the kinds of learning that help the erudite in his direct contacts with other people, then literature, history and the social sciences can claim nearly all the field. But if its boundaries are widened to include all education that directly, or somewhat indirectly, promotes social cooperation, mathematics has a large place in the sun. It is a fundamental part of every engineering project that supplies the primary social unit, the family and the communities, states and nations, with food, clothing, shelter, with protection from disease, fire, flood, with transportation and intercommunication. It has a similar part in surveys of land, in the construction and maintenance of systems of money and banking, of investments, of taxation, and of statistics of various kinds. Mathematics, then, is one of the means of transforming natural objects into social agencies.

Education is supposed to implant in the future professional man, business man, engineer, architect and tradesman an habitual attitude towards his occupation, first of all, as an essential

factor in the life of the community, second, as his means of livelihood. A surgeon's neglect to keep up with improvements in methods of performing important operations is wronging his patients much more than himself. The architect whose ignorance of a mathematical or engineering principle results in the collapse of a building with destruction of lives has proved himself remiss in his duty to society. The manufacturer who knows nothing about cost accounting must charge the users of his product unnecessarily high prices and make smaller profits. These individuals fail to appreciate their social relations. Since all adaptations are thus interwoven with man's highest obligations to man, it is neither logical nor pedagogical nor conducive to high ethical standards to label some kinds of education cultural and other kinds crass and degrading. All types of instruction can, obviously, be social, ethical, cultural.

This makes it evident that the assimilation of culture depends not so much upon the subject matter as upon the attitude of the learner, upon the purposes for which a course is given, and, above all, upon the manner in which it is taught. If a student pursues a subject only because it is in the course, with the expectation that it will be of no real use, convinced more and more that it is a mere consumer of time and energy, "busy work," it would be called in the primary grades, to keep him out of mischief and to deceive his parents into believing that their offspring is being privileged to peer into the deepest mysteries of learning, this attitude can produce nothing but disgust for all manner of schooling. This disgust is anything but culture, although it is the most common attitude in high school and college.

If a student is told that pure, uncommercialized, unapplied mathematics will in some incomprehensible way remove from his nature the remnants of the beast and the savage and make him a refined, cultured person, we cannot blame him for having his doubts. Moreover, this polished, useless type of gentleman is an unattractive ideal for our energtic American youth of today. Mathematics has enough practical applications to afford a definite illustration for nearly every assignment in each of its branches. The time consumed by stopping in the flood of letters and signs to point out a use for the thought in hand is time well spent in stimulating interest and in convincing the

student that he and his education are to be potential in human affairs.

Insistent demands for evidence that mathematics is worth the time and energy that it consumes in courses that lead away from engineering have brought forth the argument that if one learns to reason in mathematics, he can reason that much better in all fields. But it has been found through experiments that accomplishments in algebraic reasoning carry over unintentionally, unconsciously, without assistance, even into geometry in amazingly small degree, and into less related fields in more diminished amounts. It has been demonstrated also by extensive experiments that geometry is learned as easily and as well before the study of algebra as in the usual order, subsequent to algebra.

But there can be little doubt that, if similarities in the methods of reasoning in mathematics and any other field of learning were definitely pointed out to students by the teacher of mathematics or of that other field, there would be a much larger degree of transference. If the similarities and differences, not only in reasoning, but also in judging, in classifying, in perceiving, in memorizing, in imagining, in all these processes of thinking, were subjected to study from time to time, students would learn more mathematics than they do and would become better reasoners in other branches. The following illustrates how it might aid mathematics. The two most important steps in reasoning in most of the scholastic and practical realms are omitted from student work in mathematics. Look through a four-foot shelf of arithmetics and algebras and you are likely to find not more than one or two examples that contain numbers that are not needed in their solution. Selection of evidences, of facts, of data, from the mass of them that are usually present, that are really pertinent to the problem situation and necessary to its solution, is one of the most indispensable steps in thinking, a step to which textbook writers and teachers of mathematics would pay some attention if they discovered how important it is in other fields. Also if mathematicians contrasted their field carefully with other fields of learning and practical action, they would become weaned from that ancient logic that presumed that reasoning with any class of data in strict accordance with syllogistic laws would be so 'error-tight' that its results would need no testing. That old logic has been abandon-

ed because it was too cock-sure. We know today that reasonableness is scant evidence of truth. A child's first lesson in subtraction should include proof by addition, and no solution in algebra should be considered complete until the numerical value is proved to be the right one by substitution for its literal symbol. As long as selection of data and proof of results are not thoroughly habituated in mathematics, it would be better not to have its methods of reasoning carried over into other fields. It is therefore fortunate that there is little transference unless it is intentionally done.

The habit of being exact, an outstanding trait of the mathematician, is not easily brought to its desirable degree in physical and social sciences. Men with thorough mathematical training are said to acquire it more easily than others. Here again conscious intention to transfer the methods of one field to another would be worth while. In his last annual report to the Board of Trustees of Columbia University, President Butler expressed astonishment that scientific method has not found its way into practical life except in one or two departments. He remarked that eminent scientists, evidently referring to Sir Oliver Lodge and Camille Flammarion especially, seem to lock all their scientific methods in their laboratories, for they can go to a mediumistic seance and be fooled as easily as the most ignorant persons.

CURRICULUM PROBLEMS IN SECONDARY MATHEMATICS

(Concluded from the March issue)

By W. D. REEVE
Teachers College, Columbia University

Problems Relating to Content

The problems relating to content are so numerous and complicated that we can take time to discuss only a few. First, here is the question as to whether the traditional course in arithmetic, followed by algebra in the ninth grade, geometry in the tenth, and so on, is the best one to perpetuate. Perhaps we shall not want to perpetuate any plan. We have plenty of evidence in the form of widespread failures to show that the traditional organization is not satisfactory.

There are a great many of us who feel that a general

mathematics course beginning in the seventh year and extending through the high school with the Calculus as the last objective would be the best way of organizing the subject-matter in the course. The course in the regular organized junior high school is of this nature, but it is not included in most of the seventh and eighth grades, not organized on the junior high school plan. This is certainly most unfortunate. Moreover, we are not agreed upon some of the main issues. For example, we are not agreed as to whether the course should begin with arithmetic which in many cases is a review, or with intuitive geometry. Some of us think that it would be psychologically more sound to begin with the latter. Certainly we shall err if our seventh grade work in arithmetic consists mostly of drills on the fundamental skills taught in the earlier grades. The organization must be more fundamental.

Attempts have been made to set up an acceptable list of objectives[6] for the junior high school which represent what is actually being done in the schools, but we should have such objectives more widely discussed and understood.

Then there are such questions as to whether we should have a unit of demonstrative geometry in the ninth grade, whether algebra would not be better understood if it were begun in the seventh year and then scattered through the eighth and ninth years.

The senior high school course is now the least satisfactory of all. While teachers in the elementary school may still question whether the course in arithmetic is entirely satisfactory, they have improved the course greatly. We have all seen how the teachers in the junior high school have built up a progressive course. On the other hand, the teachers in the senior high school have been more or less content to let things stand as they are. The course in algebra has been improved in content, numerical trigonometry has been taught in some ninth grades, but generally speaking progress has been slow.

The more progressive teachers have been experimenting in various ways trying to perfect a better organization. Some of the schools doing such work are the Horace Mann School and the Lincoln School of Teachers College, the University High Schools of Chicago, Minneapolis, and Oakland, Cal.[7] Mr. John Swenson is doing a notable piece of work in teaching the Calculus to girls in Wadleigh High School, New York City. The read-

er is referred to a more extended article on the senior high school program in mathematics for a more complete and suggestive outline of the work in grades ten, eleven and twelve.[8]

One might raise the question as to why the study of quadratic equations should any longer be required of everybody in the ninth grade. Those who go on with the study of mathematics will have to study the topic, anyway, and those who do not continue will never have any use for the kind of work that is traditionally given.

Again, why should we attempt to give more than a one-year course of plane and solid geometry combined? No one can teach all of geometry in a lifetime and the important part of solid geometry for the well-educated citizen should not require a half-year of study. Moreover, we live in a a world of three dimensions and teach for the most part a geometry of Flatland.

One reason why we lose time in teaching mathematics is that we do not know how long it takes to obtain a desirable mastery of any topic. If we wished, we might settle this question by actual experiment. If we did this, we would not continue to do so much teaching beyond the stage of diminishing returns.

Doubtless some readers will look upon this discussion as unusually pessimistic. It was not so intended. Most of the comments are frank statements of facts. Besides, we can gain nothing by taking the attitude of the proverbial ostrich. If the reorganization of the curriculum in mathematics is necessary and improvement in instruction is possible, those best fitted to make the largest contributions are the mathematics teachers themselves. We would begin by remedying present defects. The ultimate success of our efforts will be due largely to the optimism and intelligence we show in admitting the facts and in reorganizing the course accordingly.

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[6]Smith, D. E., and Reeve, W. D., "The Teaching of Junior High School Mathematics" Chapter III. Ginn, 1927.

[7]Durst, Ethel H., "Calculus for High School" University High School Journal, University High School, Oakland, Cal.

[8]Reeve, W. D., "The Mathematics of the Senior High School." Teachers College Record, Vol. XXVII pp. 374-386.

Problems

Problem proposed by H. L. Smith. Show that the inequality

$$|(x_1-x_2)\ (y_1+y_2)+(x_2-x_3)\ (y_2+y_3)+(x_3-x_1)\ (y_3+y_1)|$$
$$\leqq |x_1-x_2|\ |y_1-y_2|+|x_2-x_3|\ |y_2-y_3|+|x_3-x_1|\ |y_3-y_1|$$

holds for all real values of the six letters involved, and give a geometric interpretation.

Problem proposed by G. M. White. Given: A circle whose diameter is d, and whose center is o.

Another circle drawn, inside the first, whose diameter is d' and whose center is o',

A line is drawn from any outside point C, cutting the two circles at a and m respectively.

A line mb is drawn making angle Cmo'=angle o'mb.

Coo' is a straight line.

Required: The distance oo' so that the sum of the lines am and mb will be a constant.

Note: This is a problem of light, mn is a convex mirror and the line Cmb is a beam of light originating at C.

If this problem has no solution as stated, can it be solved with the line Co given?

ON PARTIAL FRACTIONS

By H. L. SMITH
Louisiana State University

In the tet-books, the resolution of a given fraction into partial fractions is almost always accomplished by the method of undetermined coefficients. The coefficients are usually determined by equating the coefficients of the various powers of x on one side of a certain equation to the coefficients of like powers of x on the other side of the equation. Many books give a shorter method for determining these coefficients, but only in the case where the denominator of the given fraction can be factored into distinct linear factors. It is the purpose of this note to show how to modify and extend this method so as to apply to any case.

Let us first consider the fraction

$$F=\frac{2x^2+x-1}{(x-1)^2\ (x-2)},$$

whose denominator contains a repeated linear factor. As usual let us put $F=A/(x-1)^2+B/(x-1)+C/(x-2)$, (1)

where A, B, C are numbers to be determined. When cleared of fractions (1) becomes

$$2x^2+x-1=A(x-2)+B(x-1)(x-2)+C(x-1)^2 \quad (2)$$

Now divide both sides of (2) by $(x-1)$ and equate the reminders (by the remainder theorem this is equivalent to putting $x=1$ in (2)); the result is

$$2=-A \text{ or } A=-2.$$

Next divide both sides of (2) by $x-2$ and equate the remainders; the result is $9=C$.

It remains to find B. To do this substitute the value of A and the value of B into (2) and transpose the first and third terms of the right member of (2) to the left member:

$$2x^2+x-1+2(x-2)-9(x-1)^2=B(x-1)\ (x-2) \quad (3)$$

Finally simplify the left member of (3) and divide both sides by $(x-1)(x-2)$; the result is $-7=B$, which completes the solution of the problem.

Let us next consider the fraction

$$F=\frac{x^3-x^2+x+3}{(x^2+x+1)(x^2+2x+2)}$$

Set

$$F=(Ax+B)/(x^2+x+1)+(Cx+D)/(x^2+2x+2) \quad (4)$$

Clearing of fractions gives

$$x^3-x^2+x+3=(Ax+B)(x^2+2x+2)+(Cx+D)(x^2+x+1) \quad (5)$$

Dividing both sides by x^2+x+1 and equating remainders gives

$$2x+5=Bx+(B-A).$$

Hence $2=B$, $5=B-A$
so that $A=-3$.

Now substitute the values just found for A and B into (5) and transpose the first term on the right to the left; the result is after simpilfication,

$$4x^3+3x^2+3x-1=(Cx+D)\ (x^2+x+1).$$

Divide both sides by x^2+x+1,

$$4x-1=Cx+D,$$

which completes the solution.

It is to be noted that this method is self-checking; for an error in the determination of A and B in the first problem would resulted in non-divisibility of the left member of (3) by $(x-1)$ $(x-2)$ and the same sort of thing is true in the second. Moreover the work done justifiies the form assumed.

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